Hyperbolic Anosov C-systems. Exponential decay of correlation functions

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The uniformly hyperbolic Anosov C-systems defined on a torus have exponential instability of their trajectories, and as such C-systems have mixing of all orders and nonzero Kolmogorov entropy. The mixing property of all orders means that all its correlation functions tend to zero and the question of a fundamental interest is a speed at which they tend to zero. It was proven that the speed of decay in the C-systems is exponential, that is, the observables on the phase space become independent and uncorrelated exponentially fast. It is important to specify the properties of the C-system which quantify the exponential decay of correlations. We have found that the upper bound on the exponential decay of the correlation functions universally depends on the value of a system entropy. A quintessence of the analyses is that local and homogeneous instability of the C-system phase space trajectories translated into the exponential decay of the correlation functions at the rate which is proportional to the Kolmogorov entropy, one of the fundamental characteristics of the Anosov automorphisms. This result allows to define the decorrrelation and relaxation times of a C-system in terms of its entropy and characterise the statistical properties of a broad class of dynamical systems, including pseudorandom number generators and gravitational systems. We study statistical properties of observables defined on N-dimensional torus phase space of the Anosov C-system diffeomorthisms and specify the rate at which the exponential decay takes place. The decorrelation time and the relaxation time are inversely proportional to the entropy of the system and indicate that these time scales become shorter as entropy increases. This is an intuitively appealing result because the entropy measures the uncertainty in the description of the physical systems and here it is translated into the important time scales characteristics. As a result a perfectly deterministic dynamical system shows up a fast thermalisation and well developed statistical properties. When measuring different observables of the hyperbolic Anosov C-system it will be difficult to recognise that in reality the data are coming out from a perfectly deterministic dynamical system.

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